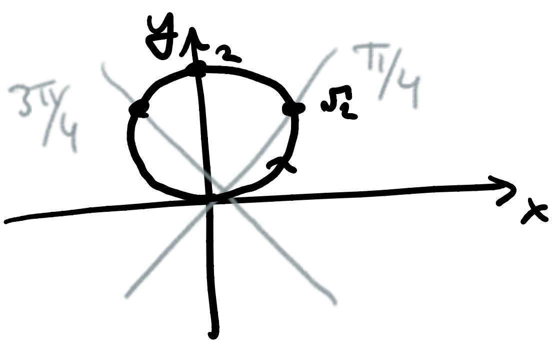
Trig Final Solutions

Tuesday, May 18, 2021 10:27 AM

Problem 1:

$$\underbrace{x^2+y^2}_{=r^2} = 2\underbrace{y}_{=r\sin\theta}$$

$$r = 2\sin\theta.$$



θ	r
0	0
$\pi/4$	$\sqrt{2}$
$\pi/2$	2
$3\pi/4$	$\sqrt{2}$
π	0

Problem 2.

$$\frac{\sin(2\theta)}{\cos\theta} + \sin^2\theta = -1.$$

$$\frac{2\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}} + \sin^2\theta = -1$$

$$\sin^2\theta + 2\sin\theta + 1 = 0$$

$$(\sin\theta + 1)^2 = 0$$

$$\sin\theta = -1$$

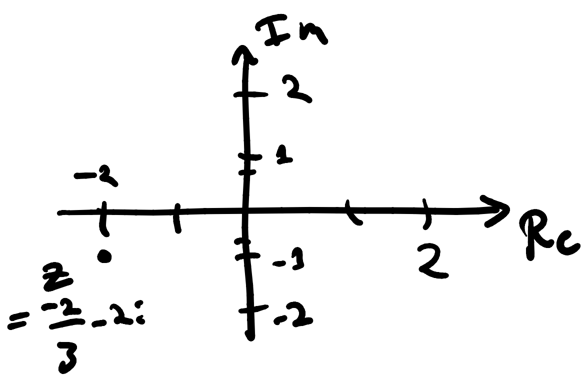
$$\boxed{\theta = \frac{3\pi}{2} + 2k\pi.}$$

Problem 3.

$$z = \frac{6-2i}{3i}$$

$$\frac{6-2i}{3i} \cdot \frac{i}{i} = \frac{6i+2}{-3}$$

$$z = \boxed{-\frac{2}{3} - 2i}$$



$$|z| = \sqrt{\left(\frac{2}{3}\right)^2 + (2)^2}$$

$$= \sqrt{\frac{4}{9} + 4}$$

$$= 2\sqrt{\frac{10}{9}}$$

$$\boxed{|z| = \frac{2}{3}\sqrt{10}}$$

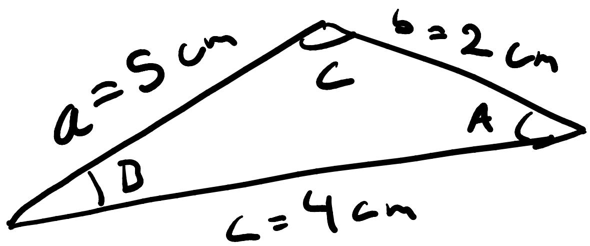
$\theta = \tan^{-1}\left(\frac{2}{\ }\right)$

argument: $\theta = \tan^{-1}\left(\frac{2}{2/3}\right)$

$$\boxed{\theta = \tan^{-1}(3)}$$

$$\boxed{z = \frac{2}{3}\sqrt{10}\left(\cos(\tan^{-1}(3)) + i\sin(\tan^{-1}(3))\right)}$$

Problem 4.



Law of Cos:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$25 = 4 + 16 - 2\cdot 4\cdot 16\cos A$$

$$\frac{5}{2\cdot 4\cdot 16} = \cos A$$

$$\boxed{A = \cos^{-1}\left(\frac{5}{8\cdot 16}\right)}$$

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin\left(\cos^{-1}\left(\frac{5}{8.16}\right)\right)}{5} = \frac{\sin B}{2}$$

$$\frac{2}{5}\left(\sqrt{1-\left(\frac{5}{8.16}\right)^2}\right) = \sin B$$

$$\boxed{B = \sin^{-1}\left(\frac{2}{5}\sqrt{1-\left(\frac{5}{8.16}\right)^2}\right).}$$

$$A + B + C = 180^\circ :$$

$$\boxed{C = 180^\circ - \sin^{-1}\left(\frac{2}{5}\sqrt{1-\left(\frac{5}{8.16}\right)^2}\right) - \cos^{-1}\left(\frac{5}{8.16}\right).}$$

Problem 5.

$$\vec{u} = \langle -1, 1 \rangle$$

$$\vec{v} = \langle 2, -2 \rangle$$

$$\frac{\vec{u}}{|\vec{u}|} = \frac{\langle -1, 1 \rangle}{\sqrt{1^2+1^2}}$$

$$= \boxed{\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 2, -2 \rangle}{\sqrt{4+4}}$$

$$\frac{v}{|\vec{v}|} = \frac{\langle 2, -2\rangle}{\sqrt{4+4}}$$

$$= \boxed{\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle}$$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

$$\langle -1, 1\rangle \cdot \langle 2, -2\rangle = \sqrt{2} \cdot \sqrt{8} \cos\theta$$

$$-2 - 2 = 4 \cos\theta$$

$$-1 = \cos\theta$$

$$\boxed{\theta = \pi}$$